

Shear Viscosity of Quark Matter

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Abstract

We consider the shear viscosity of a system of quarks and its ratio to the entropy density above the critical temperature for deconfinement. Both quantities are derived and computed for different modeling of the quark self-energy, also allowing for a temperature dependence of the effective mass and width. The behaviour of the viscosity and the entropy density is argued in terms of the strength of the coupling and of the main characteristics of the quark self-energy. A comparison with existing results is also discussed.

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I. INTRODUCTION

We address the problem of the viscosity of a Fermi system, in particular of quark matter. Much interest on this physics has been stimulated by the experiments carried out at the Relativistic Heavy Ion Collider (RHIC): these seem to indicate that above the critical temperature T_C the quark-gluon plasma (QGP) behaves as a fluid with a quite small viscosity (i.e. as an almost perfect fluid). This conclusion [1, 2, 3], mostly based on the findings on the v_2 coefficient measured [4, 5, 6, 7] in the multipole analysis of the angular distribution of the hadrons produced in an ultrarelativistic ion-ion collision, contrasts with a description of the QGP as a fluid of almost independent Landau quasi-particles which emerges from lattice calculations [8] though more recent findings [9] substantially lowered also the lattice predictions. Also the substantial collective flow (the “elliptic” flow) observed in these collisions appears to imply that the viscosity can not be that large [10, 11, 12, 13, 14]. In this work we shall examine this challenging question, confining ourselves, at present, to the quark degrees of freedom only : hence our results will not be directly comparable with the indications of the RHIC data, but should be viewed as first a step in that direction, in the absence of the gluonic degrees of freedom.

We will consider, beyond the shear viscosity η , also the ratio η/s , s being the entropy density of the system. Indeed η/s , which for vanishing chemical potential basically governs the damping rate of sound waves propagating into the system [11], has much physical significance: its temperature behaviour should determine the value T_C at which a phase transition occurs in the system [3, 15].

It should also be added that certain special supersymmetric $N = 4$ field theories (dual to black branes in higher space-time dimensions) predict a lower limit for the viscosity/entropy density ratio, namely $\eta/s \geq 1/4\pi$ (in units $\hbar = k_B = c = 1$) [16].

After introducing, in Section II, the basic formalism to derive the shear-viscosity and the entropy density starting from a dressed quark propagator (and the corresponding spectral function) we present in Section III our results and concluding remarks.

II. FORMALISM

We base our investigation, along with many authors [16, 17, 18, 19, 20], on the linear response theory which, as far as the viscosity is concerned, leads to the Kubo formula :

$$\eta(\omega) = \frac{i}{\omega} [\Pi^R(\omega) - \Pi^R(0)] \quad (1)$$

where the retarded Green's function at zero momentum is defined as

$$\Pi^R(\omega) = -i \int_0^\infty dt e^{i\omega t} \int d^3r < [T_{xy}(\mathbf{r}, t), T_{xy}(0, 0)] > \quad (2)$$

and the integrand is the correlator of the off-diagonal (x, y) element of the energy-momentum tensor $T_{\mu\nu}$ at different space-time points. In Eq. (2) the brackets imply the thermal average of the commutator.

From (1) and (2) the static ($\omega = 0$) viscosity follows:

$$\eta \equiv \eta(\omega = 0) = - \left. \frac{d}{d\omega} \text{Im } \Pi^R(\omega) \right|_{\omega=0^+} . \quad (3)$$

We conveniently get Π^R from the Matsubara formalism by analytic continuation, according to the prescription ($\delta \rightarrow 0$)

$$\Pi^R(\omega) = \Pi(i\omega_n)|_{i\omega_n=\omega+i\delta}$$

with

$$\Pi(i\omega_n) = - \int_0^\beta d\tau e^{-i\omega_n \tau} \int d^3r < T_\tau \left(T_{xy}(\mathbf{r}, \tau) T_{xy}(0, 0) \right) > ; \quad (4)$$

in the above $\omega_n = 2\pi n/\beta$, $\beta = 1/T$ and T_τ is the τ -ordered product.

Concerning the structure of the canonical energy-momentum tensor we observe that for all the Lagrangians not displaying a derivative coupling among the fields (a typical example being the Nambu–Jona-Lasinio one) it turns out that

$$T_{xy} = \frac{i}{2} (\bar{\psi} \gamma_2 \partial_1 \psi - \partial_1 \bar{\psi} \gamma_2 \psi) , \quad (5)$$

ψ being the quark field.

Following Ref. [17], in the evaluation of $\Pi(i\omega_n)$ one can stick to the first order of a ring diagram expansion, since higher orders (e.g. for a scalar or pseudoscalar interaction

coupling) vanish, in the chiral limit $m_q = 0$, due to the trace of odd numbers of γ matrices¹. Hence, by inserting (5) into (4), one easily gets:

$$\Pi(i\omega_n) = \frac{1}{\beta} \sum_l \int \frac{d^3p}{(2\pi)^3} p_x^2 \text{Tr} [\gamma_2 S(i\omega_l + i\omega_n, \mathbf{p}) \gamma_2 S(i\omega_l, \mathbf{p})] \quad (6)$$

where the trace is taken over spin, flavor and colour.

Next, by introducing the spectral representation of the quark propagator,

$$S(i\omega_l, \mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \frac{\rho(\epsilon, \mathbf{p})}{i\omega_l - \epsilon}, \quad (7)$$

with $\omega_l = (2l+1)\pi/\beta$, it is possible [17] to carry out the summation over the Matsubara frequencies via standard contour integral technique. One ends up with the following expression for the shear viscosity:

$$\eta = -\frac{1}{2} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int \frac{d^3p}{(2\pi)^3} p_x^2 \frac{\partial f}{\partial \epsilon} \text{Tr} [\rho(\epsilon, \mathbf{p}) \gamma_2 \rho(\epsilon, \mathbf{p}) \gamma_2] \quad (8)$$

where $f(\epsilon) = 1/(e^{\beta(\epsilon-\mu)} + 1)$ is the thermal distribution for fermions; we notice that the viscosity η gets its major contribution from the surface of the Fermi distribution.

In equation (8) the quark propagators are meant to be “fully” dressed within the appropriate mean-field approach, depending upon the model Lagrangian one is referring to. Here we shall adopt a merely phenomenological approach, with a suitable Ansatz for the quark self-energy.

A. The scalar case

We consider first the case of a Lorentz scalar self-energy of the form

$$\Sigma(p) = [M(p) - i\Gamma(p)] \mathbf{1} \quad (9)$$

to be inserted into the (massless) fermion propagator

$$S = \frac{1}{\not{p} - \Sigma(p)}. \quad (10)$$

¹ A different situation would be encountered by considering series of ladder diagrams [18]; however one should keep in mind that exchange-like diagrams (interaction inside a fermionic loop) are reduced with respect to the direct ones by the degeneracy of states, including the number of colors N_c .

In the above, M and Γ should be viewed as phenomenological functions of the four-momentum p , which will be specified later on.

The spectral function associated with the propagator (10) is then easily found to be

$$\rho(\epsilon, \mathbf{p}) = -i [S_A(\epsilon, \mathbf{p}) - S_R(\epsilon, \mathbf{p})] = 2 \frac{\Gamma \text{sign}(\epsilon) ((\not{p} + M)^2 + \Gamma^2)}{(p^2 - M^2 + \Gamma^2)^2 + 4M^2\Gamma^2} \quad (11)$$

where S_A and S_R are the usual advanced and retarded propagators.²

After inserting (11) into (8) and performing the relevant traces one gets the following expression for the viscosity:

$$\eta = \eta_I + \eta_{II} \quad (12)$$

with

$$\eta_I = \frac{64N_c N_f}{T} \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} (1 - f(\epsilon)) f(\epsilon) M^2 \Gamma^2 \frac{p_x^2 p_y^2}{[(p^2 - M^2 + \Gamma^2)^2 + 4M^2\Gamma^2]^2} \quad (13a)$$

and

$$\eta_{II} = -\frac{8N_c N_f}{T} \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} (1 - f(\epsilon)) f(\epsilon) \Gamma^2 \frac{p_x^2}{(p^2 - M^2 + \Gamma^2)^2 + 4M^2\Gamma^2} . \quad (13b)$$

In the above $p^2 = \epsilon^2 - \mathbf{p}^2$, N_c and N_f are the colour and flavour numbers. The temperature (and chemical potential) dependence is embedded into the Fermi distribution $f(\epsilon)$. In the similar approach by Iwasaki et al. [17] M and Γ were kept as positive, constant parameters; however with this choice η_{II} , although generally smaller than η_I if integrated up to a cutoff momentum, is divergent.

In order to ensure the convergence of η_{II} and guided by simplicity arguments, we shall consider a constant parameter M , while for the width of the quasi-particle we use:

$$\Gamma(p) = \frac{\lambda^2}{\sqrt{|\mathbf{p}|^2 + M^2}}, \quad (14)$$

λ being a constant parameter as well. One should notice that the choice of the two parameters entering into the quasi-particle self-energy is not completely arbitrary, since on the basis of general arguments the spectral function must obey the following "sum rule" [21]:

$$\frac{1}{4} \text{Tr}_{\text{Spin}} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} [\rho(\epsilon, p) \gamma_0] = 1, \quad (15)$$

² Notice that (11) differs from the Lorentzian shape one would obtain in the non-relativistic limit.

which is satisfied when the fermion propagator obeys a dispersion relation.

Let us now consider the entropy density within the same model (we stick in this paper to the $\mu = 0$ case); according to the customary field theory formulation [22] it reads:

$$\begin{aligned} s &= \frac{1}{V} \frac{\partial}{\partial T} (T \ln Z) \\ &= 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln(1 + e^{-\beta\omega_+}) + \ln(1 + e^{-\beta\omega_-}) + \frac{\beta\omega_+}{1 + e^{\beta\omega_+}} + \frac{\beta\omega_-}{1 + e^{\beta\omega_-}} \right\} \end{aligned} \quad (16)$$

where V is the normalization volume and

$$\omega_{\pm}^2(p) = \mathbf{p}^2 + M^2 - \Gamma^2 \pm 2i\Gamma M. \quad (17)$$

By separating real and imaginary parts of $\omega_{\pm}(p)$ it can be explicitly shown that s is real; one gets (in the hypothesis $\Gamma < M$):

$$s = 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln \left[e^{-2\tilde{E}_p} + 2e^{-\tilde{E}_p} \cos \tilde{\alpha}_p + 1 \right] + 2e^{-\tilde{E}_p} \frac{\tilde{E}_p (e^{-\tilde{E}_p} + \cos \tilde{\alpha}_p) + \tilde{\alpha}_p \sin \tilde{\alpha}_p}{e^{-2\tilde{E}_p} + 2e^{-\tilde{E}_p} \cos \tilde{\alpha}_p + 1} \right\} \quad (18)$$

with $\tilde{E}_p = \beta \operatorname{Re} \omega_+(p)$, $\tilde{\alpha}_p = \beta \operatorname{Im} \omega_+(p)$.

B. General self-energy

Let us now turn to a more general structure for the quark self-energy:

$$\Sigma(p) = a_0(p)\gamma^0 + a_1(p)\boldsymbol{\gamma} \cdot \mathbf{p} + a_2(p)\mathbb{1} \quad (19)$$

p being the quark four-momentum. The corresponding propagator and spectral function can be more conveniently expressed making use of the customary projector operators [23, 25]:

$$\Lambda_{\pm}(\mathbf{p}) = \frac{1}{2} \left(\mathbb{1} \pm \gamma_0 \frac{\boldsymbol{\gamma} \cdot \mathbf{p} + m}{E_p} \right) \quad (20)$$

where $E_p = \sqrt{\mathbf{p}^2 + m^2}$ and m is the bare fermion mass.

It is then easily shown that:

$$\gamma_0 S^{-1}(p) \equiv \gamma_0 (\not{p} - m - \Sigma(p)) = \Delta_+^{-1}(p) \Lambda_+(\mathbf{p}) + \Delta_-^{-1}(p) \Lambda_-(\mathbf{p}), \quad (21)$$

with

$$\Delta_{\pm}^{-1}(p_0, \mathbf{p}) = p_0 \mp [E_p + \Sigma_{\pm}(p)]. \quad (22)$$

In the above the self-energy (19) has been expressed as well in terms of the projection operators (20), with the condition $a_2(p) = ma_1(p)$:

$$\gamma_0 \Sigma(p) = \Sigma_+(p) \Lambda_+(\mathbf{p}) - \Sigma_-(p) \Lambda_-(\mathbf{p}), \quad (23)$$

where

$$\Sigma_{\pm}(p) = a_1(p) E_p \pm a_0(p). \quad (24)$$

The functions $a_i(p)$ ($i = 0, 1$) should be obtained on the basis of some microscopic calculation, thus entailing the modeling of the quark propagator in a suitable description.

Attention should be payed to the definition of the retarded and advanced propagator, for which analogous definitions hold, e.g. with

$$\Delta_{\pm}^{R(A)}(p_0, \mathbf{p}) = \frac{1}{p_0 \mp [E_p + \Sigma_{\pm}^{R(A)}(p)]}. \quad (25)$$

By definition the “retarded” self-energy must satisfy the condition:

$$\text{Im}\Sigma_+^R < 0 \quad \text{and} \quad \text{Im}\Sigma_-^R > 0 \quad (26)$$

and

$$\Sigma_{\pm}^A(p) = [\Sigma_{\pm}^R(p)]^*. \quad (27)$$

The spectral function can now be expressed as

$$\rho(p_0, \mathbf{p}) \gamma_0 = \rho_+(p_0, \mathbf{p}) \Lambda_+(\mathbf{p}) + \rho_-(p_0, \mathbf{p}) \Lambda_-(\mathbf{p}) \quad (28)$$

where

$$\rho_{\pm}(p_0, \mathbf{p}) = \mp \frac{2\text{Im}\Sigma_{\pm}^R(p)}{[p_0 \mp (E_p + \text{Re}\Sigma_{\pm}^R(p))]^2 + [\text{Im}\Sigma_{\pm}^R(p)]^2}. \quad (29)$$

With these ingredients we can now turn to the evaluation of the shear viscosity and of the entropy density; the former is obtained from formula (8) and again can be split into two terms, η_I and η_{II} , which read:

$$\eta_I = \frac{N_c N_f}{T} \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} (1 - f(\epsilon)) f(\epsilon) \frac{p_x^2 p_y^2}{E_p^2} [\rho_+(\epsilon, \mathbf{p}) - \rho_-(\epsilon, \mathbf{p})]^2 \quad (30a)$$

and

$$\eta_{II} = \frac{N_c N_f}{T} \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} (1 - f(\epsilon)) f(\epsilon) 2p_x^2 \rho_+(\epsilon, \mathbf{p}) \rho_-(\epsilon, \mathbf{p}) . \quad (30b)$$

For what concerns the entropy density, we start from the general formula [25]

$$s = -2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{(2\pi)} \frac{\partial f(\epsilon)}{\partial T} \text{Tr}_{\text{Spin}} \{ \text{Im} [\ln (-\gamma_0 S_R^{-1})] + \text{Im} (\gamma_0 \Sigma^R) \text{Re} (S_R \gamma_0) \} \quad (31)$$

the trace being taken over the spin degrees of freedom. Having expressed all relevant quantities in terms of the projector operators, the trace of the logarithm can now be carried out, yielding

$$\begin{aligned} s &= -4N_c N_f \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{(2\pi)} \frac{\partial f(\epsilon)}{\partial T} \left\{ \text{arctg} \left(\frac{-\text{Im} \Sigma_+^R}{\epsilon - (E_p + \text{Re} \Sigma_+^R)} \right) + \right. \\ &\quad + \text{arctg} \left(\frac{\text{Im} \Sigma_-^R}{\epsilon + E_p + \text{Re} \Sigma_-^R} \right) - \pi \theta[\epsilon - (E_p + \text{Re} \Sigma_+^R)] - \pi \theta[\epsilon + E_p + \text{Re} \Sigma_-^R] + \\ &\quad \left. - \frac{1}{2} \rho_+(\epsilon, p) [\epsilon - (E_p + \text{Re} \Sigma_+^R)] - \frac{1}{2} \rho_-(\epsilon, p) [\epsilon + E_p + \text{Re} \Sigma_-^R] \right\} . \end{aligned} \quad (32)$$

We now apply the above formalism to a schematic model for the quark self-energy. We stick to the chiral limit ($m = 0$) and start from the HTL self-energy [24, 25] ($p \equiv |\mathbf{p}|$):

$$\Sigma_{\pm}^{HTL}(\epsilon, p) = \frac{M^2}{p} \left(1 - \frac{\epsilon \mp p}{2p} \ln \frac{\epsilon + p}{\epsilon - p} \right) ; \quad (33)$$

then, by taking into account that at $\mu = 0$ the integrand in the shear viscosity is peaked at zero frequency, we consider the $\epsilon \rightarrow 0^+$ limit (namely $\epsilon \rightarrow i\delta$, δ being a positive infinitesimal) of the expression (33):

$$\hat{\Sigma}_{\pm}^R(p) \equiv \Sigma_{\pm}^{HTL}(0^+, p) = \frac{M^2}{p} \mp i\pi \frac{M^2}{2p} . \quad (34)$$

In the above formulas the temperature dependent mass parameter is $M = \lambda T$, where (in the original HTL formula) $\lambda^2 = C_f g^2/8 = g^2/6$ for $N_c = 3$ colors, C_f being the Casimir invariant of $SU(N_c)$ and g the strong coupling constant.

The shear viscosity is then obtained by inserting into eqs. (30a) and (30b) the following spectral densities:

$$\hat{\rho}_{\pm}(\epsilon, p) = \frac{\pi M^2}{p} \frac{1}{\left[\epsilon \mp \left(p + \frac{M^2}{p} \right) \right]^2 + \left(\frac{\pi M^2}{2p} \right)^2} . \quad (35)$$

In order to evaluate the entropy density (32) one can conveniently employ the identity

$$\frac{\partial f(\epsilon)}{\partial T} = -\frac{\partial \sigma_f(\epsilon)}{\partial \epsilon} , \quad (36)$$

with

$$\sigma_f(\epsilon) = -[f(\epsilon) \log f(\epsilon) + (1 - f(\epsilon)) \log(1 - f(\epsilon))] ; \quad (37)$$

it allows to integrate by parts over the energy in (32), yielding the simple expression³

$$s = \pi N_c N_f \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{(2\pi)} \sigma_f(\epsilon) \frac{M^2}{p} \{ \hat{\rho}_+^2(\epsilon, p) + \hat{\rho}_-^2(\epsilon, p) \} . \quad (38)$$

The function $\pi \frac{M^2}{p} \{ \hat{\rho}_+^2(\epsilon, p) + \hat{\rho}_-^2(\epsilon, p) \}$ can be interpreted as the spectral density [26] for the entropy. Notice that in getting (38) the energy independence of the self-energy (34) was crucial; moreover we removed the contributions stemming from the derivative of the theta-functions in Eq. (32), since they would have produced extra delta-peaks in the spectral density, leading to a double counting of the degrees of freedom (and altering the normalization of the spectral function itself).

III. RESULTS AND CONCLUSIONS

Using the formalism of the previous Section, we have evaluated the shear viscosity, the entropy density and their ratio with three different choices for the quark self-energy. For the simplest scalar self-energy [Eq. (9)] we first consider a constant M and Γ given by Eq. (14) with $\lambda = 0.5M$; the choice for the latter was constrained by the sum rule for the spectral density, which, for $\lambda \leq 0.5M$ is only mildly violated (e.g. with the above value it differs from 1 by less than about 10% in the interval $0 \leq p \leq 150$ MeV).

In Fig. 1 we display the result obtained for η/s with $M = 300$ MeV and 350 MeV, as a function of temperature for $T \geq 140$ MeV: the ratio grows like aT^3 , the coefficient a becoming smaller as M increases. We expect this model to be meaningful only in the regime of high temperatures, where it represents a gas of quasi-particles. It might be interesting to compare these results with the corresponding ratio obtained in a pion gas [27], also displayed in the figure. In this approach the shear viscosity is derived from a Boltzmann-Uehling-Uhlenbeck transport equation via the Chapman-Enskog method [28] to first order, while the entropy density is the local equilibrium bosonic entropy density. Since the differential cross section used in the numerical computation is the experimental one, these values of η/s at zero

³ The finite term vanishes since $\lim_{\epsilon \rightarrow \pm\infty} \sigma_f(\epsilon) = 0$.

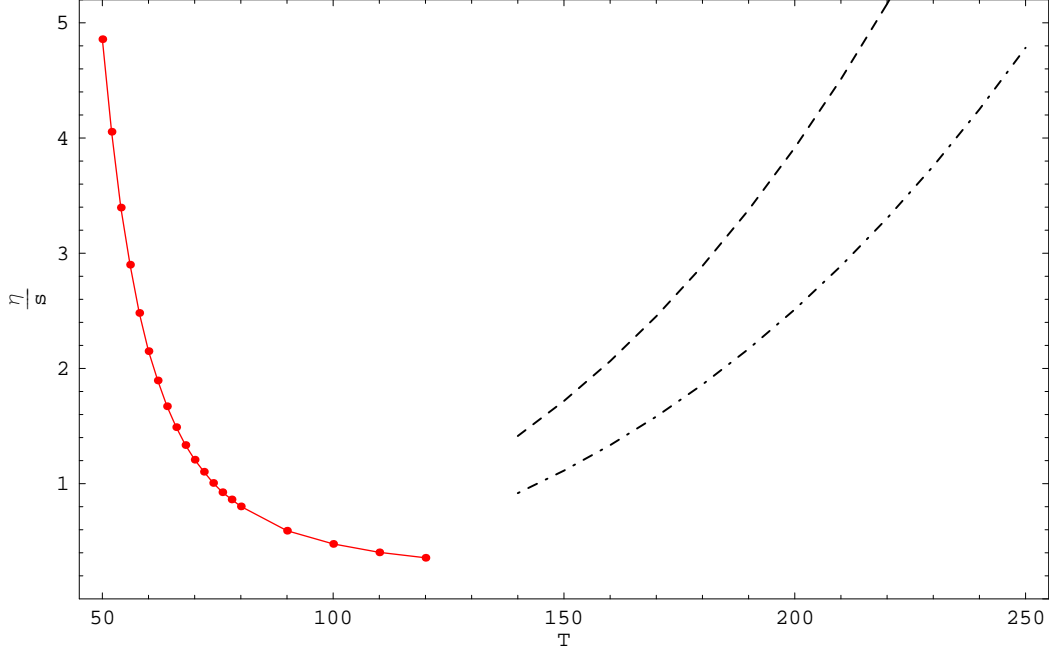


FIG. 1 The ratio η/s in the scalar model with $M = 300$ MeV (dashed line) and $M = 350$ MeV (dot-dashed line) as a function of temperature for $T > 140$ MeV; the points at low temperatures refer to the pion gas (Courtesy of D. Davesne [27]).

chemical potential are essentially model-independent. The two regimes (composite hadrons at low temperature, gas of quasi-particle quark states in the high T bath) qualitatively reproduce the analogous situation illustrated in Ref. [3], where the quark-gluon phase is described within perturbative QCD.

Within the same model, but taking into account a temperature dependence of the quasi-particle mass and width, the results for the ratio η/s drastically change: indeed by heuristically assuming the conventional thermal free mass, $M = 2\pi T$ and, in eq.(14), $\lambda = bM$, we obtained a T -independent η/s ratio, which displays a $1/b^2$ behaviour, as it is illustrated in Fig. 2. We notice that for the largest value of b reported in the figure, our η/s ratio is close to the AdS/CFT limit; moreover for $b > 0.25$ the shear viscosity becomes negative, an outcome which reflects the violation of the unitarity condition imposed by the sum rule (15); the latter indeed "protects" the positivity of η and the balance between $\text{Re}\Sigma$ and $\text{Im}\Sigma$: both grow linearly with T , but their ratio must be kept within well defined limits (with $\lambda = 0.25M$ the violation of the sum rule is smaller than 5%).

Finally we considered the self-energy of Eq. (34), which exactly satisfies the sum rule (15)

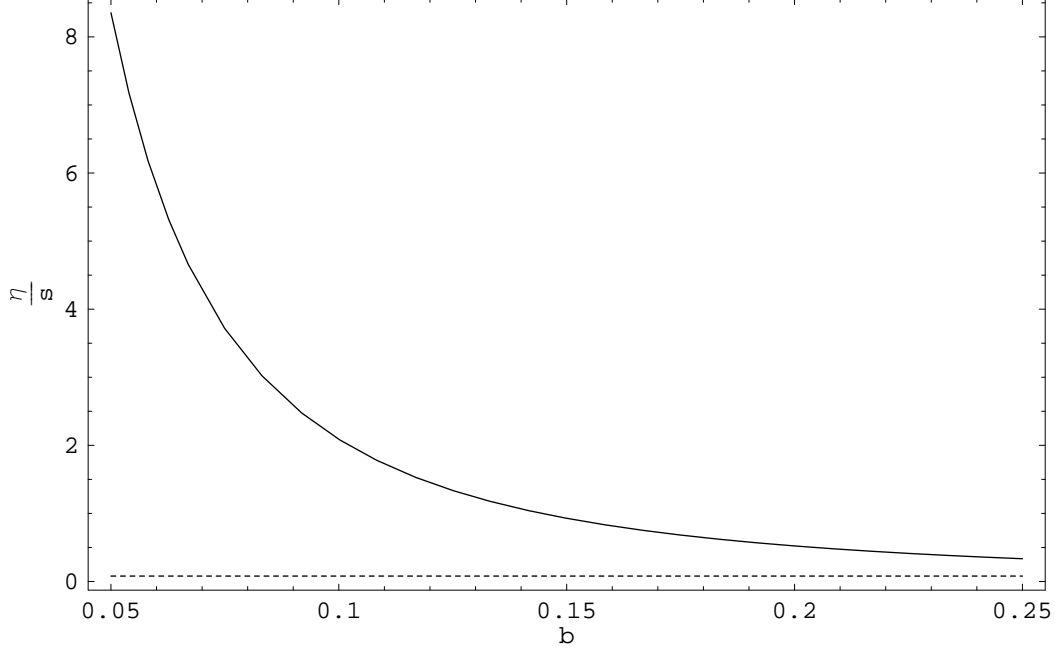


FIG. 2 Ratio η/s in the scalar model with temperature dependent mass and width as a function of $b \equiv \lambda/M$. The AdS/CFT limit is reported (dashed line).

for any choice of M . As in the previous case, we found that both the shear viscosity and the entropy density raise proportionally to T^3 , so that their ratio is temperature independent. This can be seen by dimensional analysis since, sticking to $\mu = 0$ and with a fixed strong coupling constant (namely a T -independent g), the energy scale is set by the temperature itself.

In Fig. 3 we report the quantity η/T^3 from Eqs. (30a), (30b) and s/T^3 from Eq. (38) as a function of λ , while Fig. 4 shows the corresponding ratio η/s : the latter is compared with the same quantity obtained in the scalar case, with a temperature dependent mass, the b and λ parameters being linked (at $T = 0$) by the relation $b = \lambda/\sqrt{8\pi}$. We find that in both cases η/s decreases as $1/\lambda^2$ and (since the entropy density is slightly varying with λ) the dominant trend is due to η . This $1/\Gamma$ behaviour (Γ being the particle width) is a general feature of the microscopic calculation of transport coefficients, which stems from the analytic properties of the fermion spectral function [18]; likewise in kinetic theory the same behaviour is expected since $1/\Gamma$ is related to the mean free path of particles.

Also quantitatively the results we find are compatible with previous ones. For example at $\lambda = 0.65$ ($g \simeq 1.6$) we find $\eta/T^3 = 6.7$ to be compared with $\eta/T^3 \simeq 7$ found by [29] for

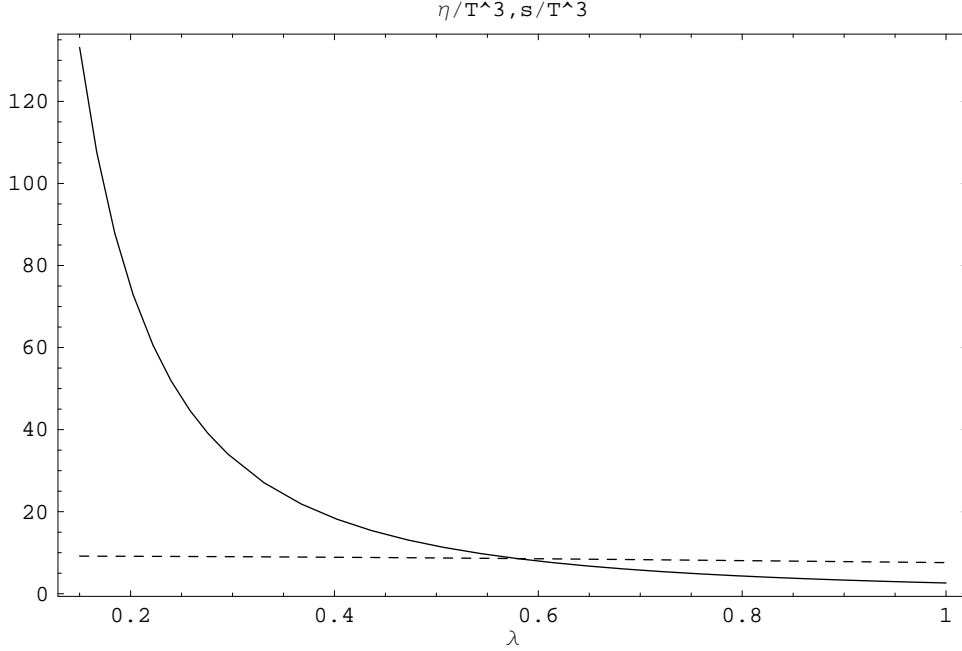


FIG. 3 The behaviour of η/T^3 (continuous line) and s/T^3 (dashed line) versus λ obtained with the self-energy (34).

a quark plasma near the critical temperature. For the same λ we get $\eta/s \simeq 0.8$ which is four times larger than the one obtained in Ref. [13] for a purely gluonic system in a similar approach. Even by pushing our model to $\lambda = 1$ we would find the somewhat larger value $\eta/s \simeq 0.4$: it seems that in the present approximation the quark contribution to η/s is larger than the gluonic one (at least if we restrict ourselves to values of λ for which the "HTL" approximation can be reasonably applied).

In spite of the different Lorentz structure of the self-energy the two ratios displayed in Fig. 4 are quite similar. We can thus infer that the relevant feature which governs the ratio η/s in the fermionic system is the temperature dependence of the effective mass and width of the quark, while the detailed (Lorentz) structure of the self-energy does not appear to be of much relevance.

In summary, we have considered in this paper the η/s ratio for a system of quarks with the purpose of investigating the impact on this quantity of the effective quark self-energy, which, although modeled on the basis of simplicity, still grasped important aspects of a realistic description. All calculations were carried out at zero chemical potential. Only for the case of a constant mass parameter we find that η/s grows with the temperature, as

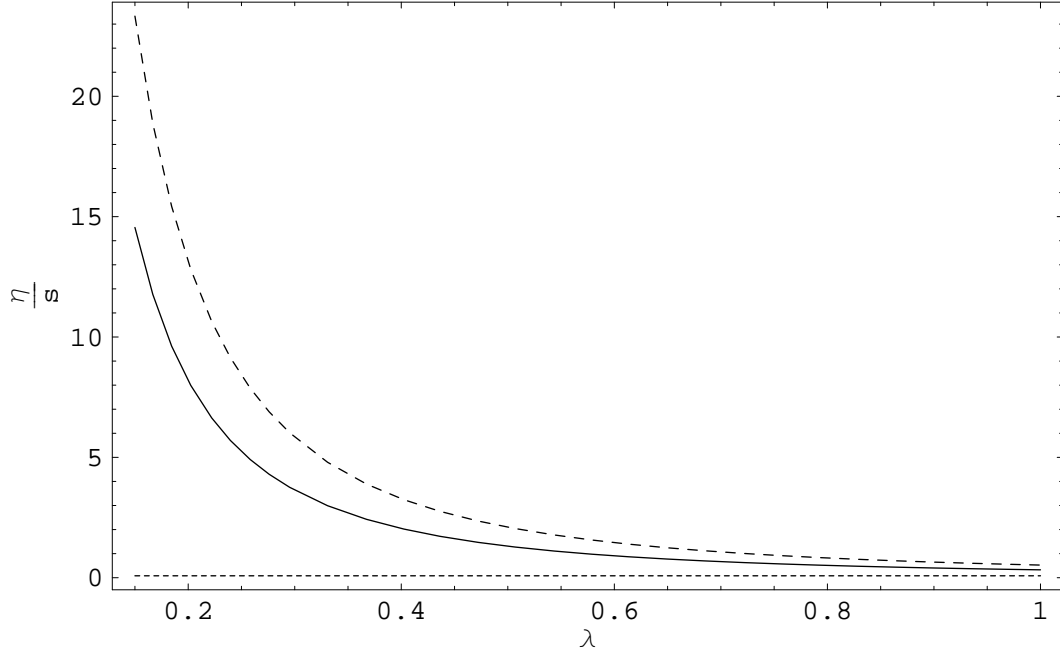


FIG. 4 The ratio η/s versus λ as obtained with the self-energy (34) (continuous line) and with the scalar, T-dependent model (dashed line); The AdS/CFT limit is also reported.

expected for a limiting case of a dilute gas system [3, 15].

On the contrary, by assuming mass and width parameters proportional to the temperature, no matter upon the details of the self-energy, the ratio η/s becomes temperature independent, since both η and s acquire a T^3 behaviour.

Concerning the specific values obtained for this ratio, in our models it obviously depends on the coupling parameter (b or λ) adopted in the definition of the quark self-energy; we found a general $1/\lambda^2$ behaviour of η/s , in agreement with previous findings. Our results are compatible with very small values of the ratio, but the fundamental requirement related to the sum rule obeyed by the quark spectral function sets some intrinsic limits on the value of the parameter itself and the AdS/CFT boundary is not reached.

In the present paper we have shown that, within relatively simple models, one can obtain a realistic estimate for η , although predictions are hindered by the arbitrariness of the employed coupling parameters. Actually, at the present stage no definite conclusion can be drawn about the shear viscosity of QGP: several microscopic models are yielding estimates of η/s , but no calculation still exists which takes into account all the relevant degrees of freedom. Moreover further indications are needed both from experiment, beyond the infor-

mation provided by v_2 [30], as well as from hydro-calculations [31, 32]. Also the behaviour of this quantity in the hadronic phase [33, 34] deserves further investigation in order to get quantitative information on the deconfinement phase transition.

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